

Name: _____ Date: _____ Class: _____

ESSENTIAL QUESTION

How does the focus-directrix definition of a parabola lead to the standard form equation, and how does the value of p determine the shape and direction of the parabola?

Lesson Overview

A parabola is the set of all points equidistant from a fixed point (the focus) and a fixed line (the directrix): $PF = PD$. The value p is the directed distance from the vertex to the focus, and it controls both the direction the parabola opens and how wide or narrow it is.

$$\begin{aligned} x^2 &= 4py \text{ (vertical)} \\ y^2 &= 4px \text{ (horizontal)} \\ (x-h)^2 &= 4p(y-k) \text{ or } (y-k)^2 = 4p(x-h) \end{aligned}$$

Vertical	$x^2 = 4py$, vertex $(0,0)$, focus $(0,p)$, directrix $y = -p$. Opens up if $p > 0$, down if $p < 0$.
Horizontal	$y^2 = 4px$, vertex $(0,0)$, focus $(p,0)$, directrix $x = -p$. Opens right if $p > 0$, left if $p < 0$.
Vertex (h,k)	Vertical: $(x-h)^2 = 4p(y-k)$, focus $(h,k+p)$. Horizontal: $(y-k)^2 = 4p(x-h)$, focus $(h+p,k)$.
Latus Rectum	The chord through the focus perpendicular to the axis; length = $ 4p $.
Key Rule	x^2 term $\rightarrow$ vertical parabola (opens up/down). y^2 term $\rightarrow$ horizontal parabola (opens left/right).

Worked Examples**EXAMPLE 1**

Identify all features of $x^2 = 12y$.

- Form: $x^2 = 4py \rightarrow 4p = 12 \rightarrow p = 3$
- Opens upward ($p > 0$)
- Vertex: $(0,0)$; Axis of symmetry: $x = 0$; Latus rectum length: $|4p| = 12$

Answer: Vertex $(0,0)$, focus $(0,3)$, directrix $y = -3$, opens up

EXAMPLE 2

Write the equation of a parabola with vertex $(0,0)$ and focus $(-4, 0)$.

- Focus on x -axis $\rightarrow$ horizontal parabola
- $p = -4$ (negative $\rightarrow$ opens left)
- Form: $y^2 = 4px = 4(-4)x = -16x$

Answer: $y^2 = -16x$

EXAMPLE 3

Find all features of $(x-2)^2 = 8(y+1)$.

- Vertex: $(2, -1)$; $4p = 8 \rightarrow p = 2$ (opens upward)
- Focus: $(2, -1+2) = (2, 1)$
- Directrix: $y = -1-2 = -3$; Axis of symmetry: $x = 2$

Answer: Vertex $(2, -1)$, focus $(2, 1)$, directrix $y = -3$

EXAMPLE 4

Write the equation of a parabola with vertex $(3, -2)$ and directrix $x = 1$.

- Directrix is vertical $\rightarrow$ horizontal parabola
- Distance from vertex to directrix: $|3-1| = 2 \rightarrow p = 2$ (opens right, since focus is to the right)
- Form: $(y-(-2))^2 = 4(2)(x-3) \rightarrow (y+2)^2 = 8(x-3)$

Answer: $(y+2)^2 = 8(x-3)$

EXAMPLE 5

Convert $y^2 - 6y - 8x + 1 = 0$ to standard form.

- Isolate y terms: $y^2 - 6y = 8x - 1$
- Complete the square: $(y-3)^2 - 9 = 8x - 1 \rightarrow (y-3)^2 = 8x + 8 = 8(x+1)$
- Vertex: $(-1, 3)$; $4p = 8 \rightarrow p = 2$; opens right

Answer: $(y-3)^2 = 8(x+1)$; vertex $(-1, 3)$, focus $(1, 3)$, directrix $x = -3$

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

- 1** Identify all features of $y^2 = -20x$.

Hint: Form $y^2 = 4px$ with $4p = -20$. Negative p means opens left.

- 2** Write the equation of a parabola with vertex $(0, 0)$ and directrix $y = 5$.

Hint: Directrix $y = 5$ is above the vertex $\rightarrow$ parabola opens downward. $p = -5$.

- 3** Find all features of $(y+4)^2 = -12(x-1)$.

Hint: Horizontal parabola. $4p = -12 \rightarrow p = -3$. Opens left.

- 4** Write the equation of a parabola with focus (2, 3) and directrix $y = -1$.

Hint: Vertex is midpoint between focus and directrix: $y\text{-coordinate} = (3 + (-1))/2 = 1$. $p = 3 - 1 = 2$.

- 5** Convert $x^2 + 4x - 8y + 20 = 0$ to standard form.

Hint: Complete the square for x : $(x+2)^2 = 8y - 16 = 8(y-2)$.

Key Vocabulary

Parabola	Set of all points equidistant from the focus and the directrix.
Focus	The fixed point inside the parabola; distance p from vertex.
Directrix	The fixed line outside the parabola; distance p from vertex on the opposite side.
Vertex	The point on the parabola closest to the directrix; midpoint between focus and directrix.
p value	The directed distance from vertex to focus; $p > 0$ opens up/right, $p < 0$ opens down/left.
Latus rectum	The chord through the focus perpendicular to the axis; length = $ 4p $.

Quick Check Quiz

Circle the letter of the correct answer for each question.

- 1** For $x^2 = -8y$, the focus is at:

Multiple Choice

A (0, 2)

B (0, -2)

C (2, 0)

D (-2, 0)

- 2** A parabola has vertex (0,0) and focus (0, 5). Its equation is:

Multiple Choice

A $x^2 = 5y$

B $x^2 = 20y$

C $y^2 = 20x$

D $x^2 = -20y$

- 3** For $(x-3)^2 = -12(y+1)$, the directrix is:

Multiple Choice

A $y = 2$

B $y = -4$

C $y = -1$

D $x = 3$

4 The latus rectum length of $y^2 = 16x$ is:

Multiple Choice

A 4

B 8

C 16

D 32

5 Which parabola opens to the left?

Multiple Choice

A $x^2 = 4y$

B $y^2 = 4x$

C $y^2 = -4x$

D $x^2 = -4y$

Common Mistakes to Avoid

✗ Confusing the sign of p with the direction of opening.

✓ $p > 0$ opens up (vertical) or right (horizontal).
 $p < 0$ opens down (vertical) or left (horizontal).
 The sign of p tells you the direction.

✗ Placing the focus and directrix on the same side of the vertex.

✓ The focus is INSIDE the parabola; the directrix is OUTSIDE. They are on OPPOSITE sides of the vertex, each at distance $|p|$.

✗ Using the wrong form for horizontal vs. vertical parabolas.

✓ If x is squared $\rightarrow$ vertical parabola (opens up/down). If y is squared $\rightarrow$ horizontal parabola (opens left/right).

✗ Forgetting to divide by 4 to find p from the equation $x^2 = 4py$.

✓ Always write the equation as $x^2 = 4py$ first, then $p = (\text{coefficient})/4$. Don't confuse $4p$ with p .

Math Tips

- ★ Quick identification: $x^2 \rightarrow$ vertical parabola; $y^2 \rightarrow$ horizontal parabola.
- ★ The vertex is always the midpoint between the focus and the directrix.
- ★ Latus rectum = $|4p|$. A wider parabola has a larger $|p|$; a narrower one has a smaller $|p|$.
- ★ To find p from standard form: $4p = \text{coefficient of the linear term}$. Then $p = \text{coefficient}/4$.
- ★ When completing the square for a parabola, move the linear term to the right side FIRST, then complete the square on the quadratic side.